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Prob #	1	2	3	4	Total
Points	18	21	21	40	

Time: 80 Minutes

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$$F(\mathbf{x}) = F(\mathbf{x}^*) + \nabla F(\mathbf{x})^T \Big|_{\mathbf{x} = \mathbf{x}^*} (\mathbf{x} - \mathbf{x}^*) \\ + \frac{1}{2} (\mathbf{x} - \mathbf{x}^*)^T \nabla^2 F(\mathbf{x}) \Big|_{\mathbf{x} = \mathbf{x}^*} (\mathbf{x} - \mathbf{x}^*) + \dots$$

$$\frac{\mathbf{p}^T \nabla F(\mathbf{x})}{\|\mathbf{p}\|} \quad \frac{\mathbf{p}^T \nabla^2 F(\mathbf{x}) \mathbf{p}}{\|\mathbf{p}\|^2} \quad \alpha_k = -\frac{\mathbf{g}_k^T \mathbf{p}_k}{\mathbf{p}_k^T \mathbf{A} \mathbf{p}_k}$$

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \alpha_k \mathbf{g}_k \quad \mathbf{x}_{k+1} = \mathbf{x}_k + \alpha_k \mathbf{p}_k$$

$$L_i = \sum_{j \neq i} \max(0, y_j - y_i + \Delta)$$

$$S(y_i) = \frac{e^{y_i}}{\sum_j e^{y_j}}$$

$$H(p,q)=-\sum_x p(x)\log(q(x))$$

$$L_i = -\log\left(\frac{e^{y_i}}{\sum_j e^{y_j}}\right)$$

$$KL\ Divergence = -\frac{1}{2}\sum (1 + \log(\sigma^2) - \mu^2 - \sigma^2)$$

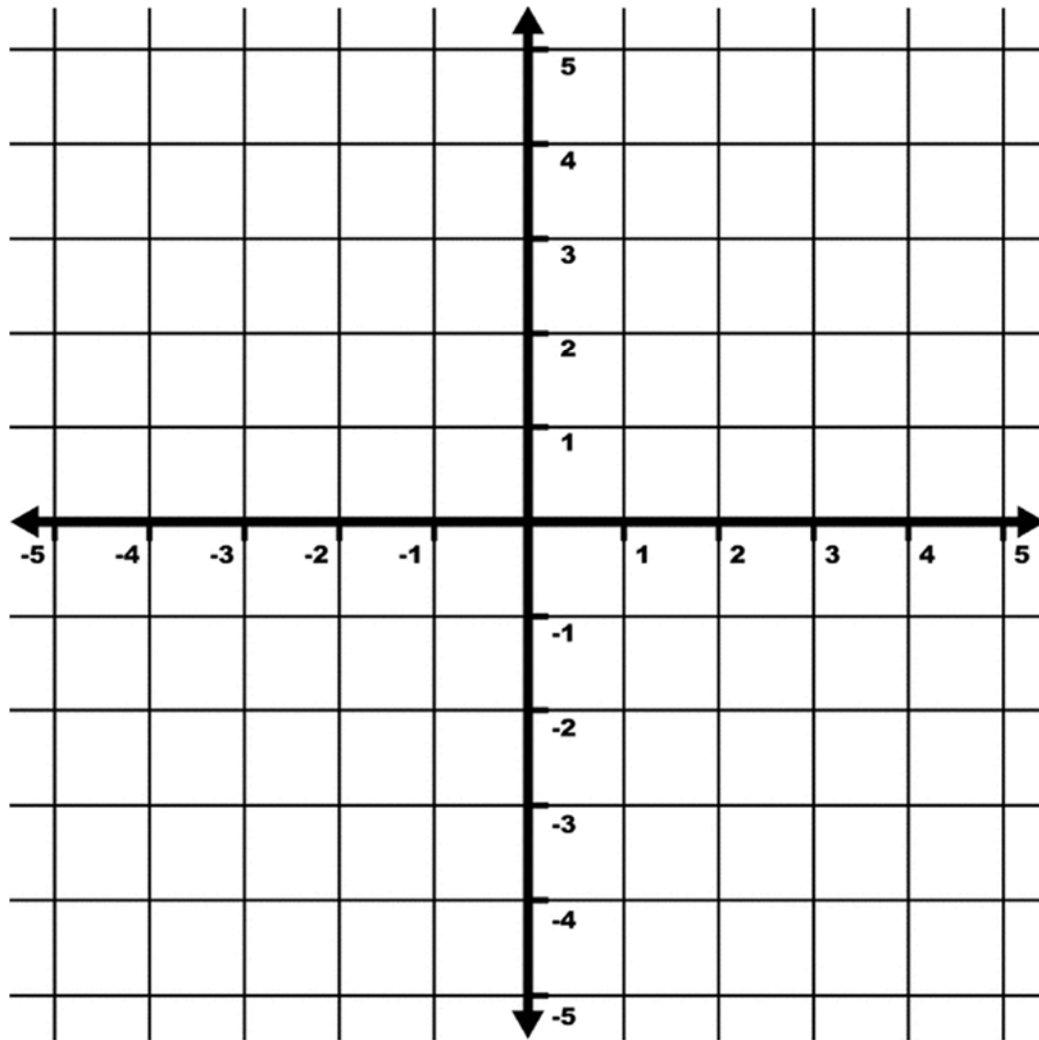
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1. Consider the following data where p is the input and t is the desired output

$$\{p_1 = \begin{bmatrix} 4.5 \\ 0 \end{bmatrix}, t_1 = 0\}, \{p_2 = \begin{bmatrix} -3.5 \\ 0 \end{bmatrix}, t_2 = 0\}, \{p_3 = \begin{bmatrix} 0 \\ 2.5 \end{bmatrix}, t_3 = 0\},$$

$$\{p_4 = \begin{bmatrix} 3.5 \\ 0 \end{bmatrix}, t_4 = 1\}, \{p_5 = \begin{bmatrix} -2.5 \\ 0 \end{bmatrix}, t_5 = 1\}, \{p_6 = \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}, t_6 = 1\}$$

DESIGN a two-layer Perceptron neural network which will correctly classify the input data. Assume the activation function for all the nodes are hardlimit with the output of 0 and 1. Show the weight matrices and biases for both layers. Biases should be included in the weight matrix in the first column.



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Problem 1 Continued

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2. Consider a convolutional neural network.

Note: **Do NOT consider Biases.**

Input layer:

Input to this CNN are color images of size **67x67x3** with the **batch size = 90**

Next layer is Conv2D layer:

Number of filters: **20**, filter size: **7x7**; stride: **2x2**; padding: **2x2**

Q1: What is the shape of the weight matrix for this layer? **Q1:** _____

Q2: What is the shape of the output (tensor) of this layer? **Q2:** _____

Next layer is Conv2D layer:

Number of filters: **30**, filter size: **5x5**; stride: **3x3**; padding: **4x4**

Q3: What is the shape of the weight matrix for this layer? **Q3:** _____

Q4: What is the shape of the output (tensor) of this layer? **Q4:** _____

Next layer is Flatten layer:

Q5: What is the shape of the output (tensor) for this layer? **Q5:** _____

Next layer is Dense layer:

Number of nodes: **50**

Q6: What is the shape of the weight matrix for this layer? **Q6:** _____

Q7: What is the shape of the output (tensor) for this layer? **Q7:** _____

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Problem 2 Continued

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3. Assuming that the actual output and the desired output of a neural network are given.

Complete the code for the following function to calculate the **mean cross entropy** loss for a batch of data.

Note:

Assume that actual output and desired output are logits. This means you have to apply **softmax** to both **actual outputs** and **desired outputs** before cross entropy calculation.

Notes:

Only use numpy. Do NOT use PyTorch or any other package.

You may use the numpy helper functions such as `np.log()` and `np.sum()`

```
def calculate_mean_cross_entropy_loss(y_hat,y):
```

```
    Import numpy as np
```

```
    """ This function calculates the mean cross entropy loss.
```

```
    :param y_hat: actual output [number_of_samples,number_of_classes].
```

```
    :param Y: Desired output [number_of_samples,number_of_classes].
```

```
    :return: mean cross-entropy loss
```

```
    """
```


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4. Complete the following function. This function should initialize a variational autoencoder and return the mean loss over all the samples. **Only use numpy.**

Notes:

Ignore biases. Initialize all weights to **ones**. Assume all layers are **fully connected**.

Activation function for layers are **Sigmoid**.

Loss is: **MSE + KL divergence**

Decoder has the same structure as the encoder in reverse order. #

Complete the incomplete functions

```
import numpy as np

def variational_autoencoder(input, encoder_layers):
    # input: numpy array of inputs [nof_train_samples, input_dimensions]
    # encoder_layers: list of integers representing number of nodes in
    # each layer of the encoder. The last number represents the dimension of latent
    # space
    # return: mean_loss

    # Step 1 (8 points)
    list_of_encoder_weights, list_of_decoder_weights =
initialize_encoder_and_decoder_layers(encoder_layers, input)
    # Step 2 (8 points)
    mu, var = encoder_forward_pass(input, list_of_encoder_weights)
    # Step 3 (8 points)
    z = sampling_trick(mu, var)
    # Step 4 (8 points)
    reconstructed_input = decoder_forward_pass(z, list_of_decoder_weights)
    # Step 5 (8 points)
    mean_loss = calculate_loss(input, reconstructed_input, mu, sigma)
    return mean_loss
```


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Problem 4 Continued

```
def decoder_forward_pass(z, list_of_decoder_weights):  
#z:numpy array of latent variables [nof_train_samples,latent_dimension]  
# return reconstructed_input [nof_train_samples, input_dimensions]
```

```
def calculate_loss(input, reconstructed_input, mu, sigma):  
# input: numpy array of inputs [nof_train_samples, input_dimensions]  
# return mean_loss (MSE + KL divergence) it should be a single number
```
